

# Phenomenological Analysis of $D$ Meson Lifetimes

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## Abstract

The QCD-based operator-product-expansion technique is systematically applied to the study of charmed meson lifetimes. We stress that it is crucial to take into account the momentum of the spectator light quark of charmed mesons, otherwise the destructive Pauli-interference effect in  $D^+$  decays will lead to a negative decay width for the  $D^+$ . We have applied the QCD sum rule approach to estimate the hadronic matrix elements of color-singlet and color-octet 4-quark operators relevant to nonleptonic inclusive  $D$  decays. The lifetime of  $D_s^+$  is found to be longer than that of  $D^0$  because the latter receives a constructive  $W$ -exchange contribution, whereas the hadronic annihilation and leptonic contributions to the former are compensated by the Pauli interference. We obtain the lifetime ratio  $\tau(D_s^+)/\tau(D^0) \approx 1.08 \pm 0.04$ , which is larger than some earlier theoretical estimates, but still smaller than the recent measurements by CLEO and E791.

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## I. INTRODUCTION

It is well known that the observed lifetime difference between the  $D^+$  and  $D^0$  is ascribed to the destructive interference in  $D^+$  decays and/or the constructive  $W$ -exchange contribution to  $D^0$  decays (for a review, see e.g., [1]). By contrast, the  $D_s^+$  and  $D^0$  lifetimes are theoretically expected to be close to each other. For example, it is estimated in [2] that

$$\frac{\tau(D_s^+)}{\tau(D^0)} = 1.00 - 1.07. \quad (1.1)$$

However, the recent Fermilab E791 measurement of the  $D_s^+$  lifetime yields  $\tau(D_s^+) = 0.518 \pm 0.014 \pm 0.007$  ps [3]. When combining with the world average  $D^0$  lifetime [4] yields the ratio

$$\frac{\tau(D_s^+)}{\tau(D^0)} = 1.25 \pm 0.04 \quad (\text{E791}), \quad (1.2)$$

which is different from unity by  $6\sigma$ . Meanwhile, the CLEO measurement of  $D_s^+$  and  $D^0$  lifetimes indicates  $\tau(D_s^+) = 0.4863 \pm 0.015 \pm 0.005$  ps [5] and

$$\frac{\tau(D_s^+)}{\tau(D^0)} = 1.19 \pm 0.04 \quad (\text{CLEO}), \quad (1.3)$$

which is  $5\sigma$  different from unity. Note that the  $D_s^+$  lifetime measured by Fermilab and CLEO is better than the errors of the world average value [4] and that the lifetime ratio of  $D_s^+$  to  $D^0$  is larger than the previous world average [4]:

$$\frac{\tau(D_s^+)}{\tau(D^0)} = 1.13 \pm 0.04 \quad (\text{PDG}). \quad (1.4)$$

Based on the operator product expansion (OPE) approach for the analysis of inclusive weak decays of heavy hadrons, it is known that the  $1/m_c^2$  corrections due to the nonperturbative kinetic and chromomagnetic terms are small and essentially canceled out in the lifetime ratios. By contrast, the  $1/m_c^3$  corrections due to 4-quark operators can be quite significant because of the phase-space enhancement by a factor of  $16\pi^2$ . The nonspectator effects of order  $1/m_c^3$  involve the Pauli interference in  $D^+$  decay, the  $W$ -exchange in  $D^0$  decay, the  $W$ -annihilation and Cabibbo-suppressed Pauli interference in nonleptonic  $D_s^+$ . While the semileptonic decay rates of  $D^+$ ,  $D^0$  and  $D_s^+$  are essentially the same, there is an additional purely leptonic decay contribution to  $D_s^+$ , namely  $D_s^+ \rightarrow \tau\bar{\nu}$ . The dimension-6 four-quark operators which describe the nonspectator effects in inclusive decays of heavy hadrons are well known [6,7]. However, it is also known that there is a serious problem with the evaluation of the destructive Pauli interference  $\Gamma^{\text{int}}(D^+)$  in  $D^+$ . A direct calculation indicates that  $\Gamma^{\text{int}}(D^+)$  overcomes the  $c$  quark decay rate so that the resulting nonleptonic decay width of  $D^+$  becomes negative [8,9]. This certainly does not make sense. This implies that the  $1/m_c$  expansion is not well convergent and sensible, to say the least. In other words, higher dimension terms are in principle also important. It has been conjectured in [8] that higher-dimension corrections amount to replacing  $m_c$  by  $m_D$  in the expansion parameter

$f_D^2 m_D / m_c^3$ , so that it becomes  $f_D^2 / m_D^2$ . As a consequence, the destructive Pauli interference will be reduced by a factor of  $(m_c / m_D)^3$ .

Another way of alleviating the problem is to realize that the usual local four-quark operators are derived in the heavy quark limit so that the effect of spectator light quarks can be neglected. Since the charmed quark is not heavy enough, it is very important, as stressed by Chernyak [9], for calculations with charmed mesons to account for the nonzero momentum of spectator quarks. It turns out that the Pauli interference in  $D^+$  decay is suppressed by a factor of  $(\langle p_c \rangle - \langle p_d \rangle)^2 / \langle p_c \rangle^2 = (\langle p_D \rangle - 2\langle p_d \rangle)^2 / m_c^2$ , where  $\langle p_c \rangle$  and  $\langle p_d \rangle$  are the momenta of the  $c$  and  $\bar{d}$  quarks, respectively, in the  $D^+$  meson. Because the charmed quark is not heavy, the spectator  $\bar{d}$  quark carries a sizable fraction of the charmed meson momentum. Consequently, the Pauli effect in  $D^+$  decay is subject to a large suppression and will not overcome the leading  $c$  quark decay width. Based on this observation, in the present paper we will follow [9] to take into account the effects of the spectator quark's momentum consistently. In the framework of heavy quark expansion, this spectator effect can be regarded as higher order  $1/m_c$  corrections.

In order to understand the  $D$ -meson lifetime pattern, it is important to have a reliable estimate of the hadronic matrix elements. In the present paper we will employ the QCD sum rule to evaluate the unknown hadronic parameters  $B_1, B_2, \varepsilon_1, \varepsilon_2$ , to be introduced below. In Sec. II, we will outline the general framework for the study of the charmed meson lifetimes. Then in Sec. III we proceed to compute the hadronic parameters using the sum rule approach. Sec. IV presents results and discussions.

## II. GENERAL FRAMEWORK

The inclusive nonleptonic and semileptonic decay rates of a charmed meson to order  $1/m_c^2$  are given by [6,7]

$$\Gamma_{\text{NL,spec}}(D) = \frac{G_F^2 m_c^5}{192\pi^3} N_c V_{\text{CKM}}^2 \frac{1}{2m_D} \left\{ \left( c_1^2 + c_2^2 + \frac{2c_1 c_2}{N_c} \right) - \left[ \alpha I_0(x, 0, 0) \langle D | \bar{c}c | D \rangle - \frac{1}{m_c^2} I_1(x, 0, 0) \langle D | \bar{c}g_s \sigma \cdot Gc | D \rangle \right] - \frac{4}{m_c^2} \frac{2c_1 c_2}{N_c} I_2(x, 0, 0) \langle D | \bar{c}g_s \sigma \cdot Gc | D \rangle \right\}, \quad (2.1)$$

where  $\sigma \cdot G = \sigma_{\mu\nu} G^{\mu\nu}$ ,  $x = (m_s / m_c)^2$ ,  $N_c$  is the number of colors, the parameter  $\alpha$  denotes QCD radiative corrections [10], and

$$\Gamma_{\text{SL}}(D) = \frac{G_F^2 m_c^5}{192\pi^3} |V_{cs}|^2 \frac{\eta(x, x_\ell, 0)}{2m_D} \times \left[ I_0(x, 0, 0) \langle D | \bar{c}c | D \rangle - \frac{1}{m_c^2} I_1(x, 0, 0) \langle D | \bar{c}g_s \sigma \cdot Gc | D \rangle \right], \quad (2.2)$$

where  $\eta(x, x_\ell, 0)$  with  $x_\ell = (m_\ell / m_Q)^2$  is the QCD radiative correction to the semileptonic decay rate and its general analytic expression is given in [11]. In Eqs. (2.1) and (2.2),  $I_{0,1,2}$  are phase-space factors (see e.g. [12] for their explicit expressions), and the factor  $V_{\text{CKM}}$  takes

care of the relevant Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. In Eq. (2.1)  $c_1$  and  $c_2$  are the Wilson coefficients in the effective Hamiltonian.

The two-body matrix elements in Eqs. (2.1) and (2.2) can be parameterized as

$$\begin{aligned}\frac{\langle D|\bar{c}c|D\rangle}{2m_D} &= 1 - \frac{K_D}{2m_c^2} + \frac{G_D}{2m_c^2} + O(1/m_c^3), \\ \frac{\langle D|\bar{c}\frac{1}{2}g_s\sigma\cdot Gc|D\rangle}{2m_D} &= G_D + O(1/m_c),\end{aligned}\quad (2.3)$$

where

$$\begin{aligned}K_D &\equiv -\frac{\langle D|\bar{h}_v^{(c)}(iD_\perp)^2h_v^{(c)}|D\rangle}{2m_D} = -\lambda_1, \\ G_D &\equiv \frac{\langle D|\bar{h}_v^{(c)}\frac{1}{2}g_s\sigma\cdot Gh_v^{(c)}|D\rangle}{2m_D} = 3\lambda_2.\end{aligned}\quad (2.4)$$

The nonperturbative parameter  $\lambda_2$  is obtained from the mass squared difference of the vector and pseudoscalar mesons:

$$\begin{aligned}(\lambda_2)_D &= \frac{3}{4}(m_{D^*}^2 - m_D^2) = 0.138 \text{ GeV}^2, \\ (\lambda_2)_{D_s} &= \frac{3}{4}(m_{D_s^*}^2 - m_{D_s}^2) = 0.147 \text{ GeV}^2.\end{aligned}\quad (2.5)$$

As for the parameter  $\lambda_1$ , it is determined from the mass relation [2]

$$(\lambda_1)_{D_s} - (\lambda_1)_D \cong \frac{2m_b m_c}{m_b - m_c} [\bar{m}_{B_s} - \bar{m}_B - (\bar{m}_{D_s} - \bar{m}_D)], \quad (2.6)$$

where  $\bar{m}_P = \frac{1}{4}(m_P + 3m_{P^*})$  denotes the spin-averaged meson mass. For  $m_b = 5.05 \text{ GeV}$  and  $m_c = 1.65 \text{ GeV}$ , we obtain  $(\lambda_1)_{D_s} - (\lambda_1)_D = -0.067 \text{ GeV}^2$ .

To the order of  $1/m_c^3$ , the nonspectator effects due to the Pauli interference and  $W$ -exchange (see Fig. 1) may contribute significantly to the lifetime ratios due to the two-body phase-space enhancement by a factor of  $16\pi^2$  relative to the three-body phase space for heavy quark decay. As stressed in the Introduction, it is crucial to invoke the effect of the light quark's momentum in the charmed meson in order to properly describe the  $D$  lifetimes. For this purpose, the four-quark operators relevant to inclusive nonleptonic  $D$  decays are [9]

$$\begin{aligned}\mathcal{L}_{\text{NL,nspec}} &= \frac{2G_F^2}{\pi} V_{\text{CKM}} \left\{ g^{\mu\nu} k^2 \eta_1 \left[ \left( 2c_1 c_2 + \frac{1}{N_c} (c_1^2 + c_2^2) \right) O_{\mu\nu}^d + 2(c_1^2 + c_2^2) T_{\mu\nu}^d \right] \right. \\ &\quad + \frac{1}{3} (k^\mu k^\nu \eta_2 - k^2 g^{\mu\nu} \eta_3) \left[ N_c \left( c_2 + \frac{1}{N_c} c_1 \right)^2 O_{\mu\nu}^u + 2c_1^2 T_{\mu\nu}^u \right. \\ &\quad \left. \left. + N_c \left( c_1 + \frac{1}{N_c} c_2 \right)^2 O_{\mu\nu}^s + 2c_2^2 T_{\mu\nu}^s \right] \right\},\end{aligned}\quad (2.7)$$

where

$$\begin{aligned}O_{\mu\nu}^q &= \bar{c}_L \gamma_\mu q_L \bar{q}_L \gamma_\nu c_L, \\ T_{\mu\nu}^q &= \bar{c}_L \gamma_\mu t^a q_L \bar{q}_L \gamma_\nu t^a c_L,\end{aligned}\quad (2.8)$$

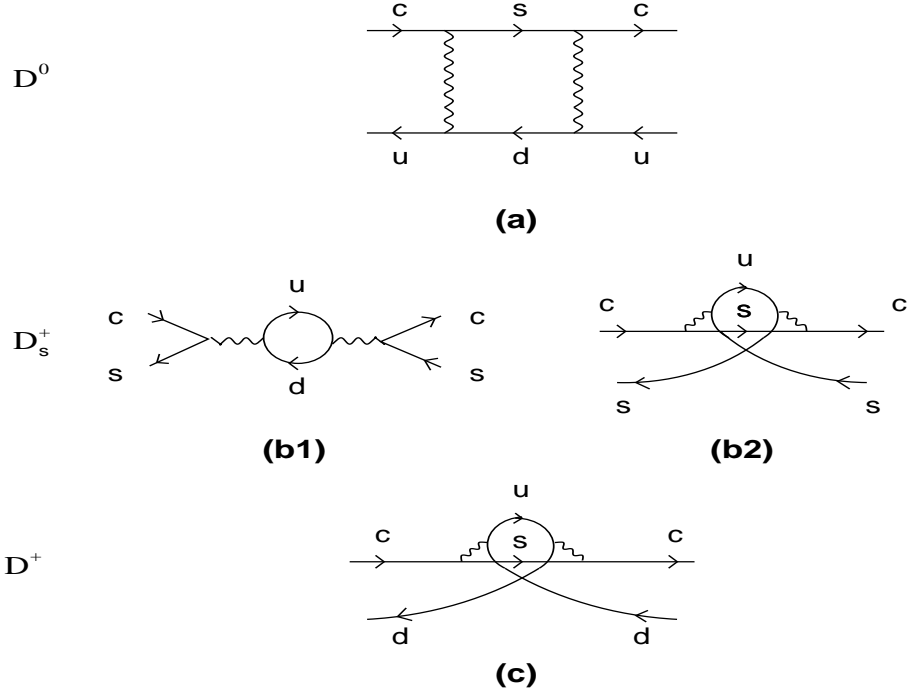


FIG. 1. Nonspectator effects: (a)  $W$ -exchange, (b1)  $W$ -annihilation, (b2) and (c) Pauli interference.

with  $t^a = \lambda^a/2$  and  $\lambda^a$  being the Gell-Mann matrices, and  $\eta_1, \eta_2, \eta_3$  are phase-space factors, depending on the number of strange quarks inside the loop of Fig. 1 [9,13]:

$$\begin{aligned}
 (i) \quad & \eta_1 = (1-x)^2, \quad \eta_2 = (1-x)^2(1+\frac{x}{2}), \quad \eta_3 = (1-x)^2(1+2x), \\
 (ii) \quad & \eta_1 = (1-x)^2, \quad \eta_2 = \sqrt{1-4x}(1-x), \quad \eta_3 = \sqrt{1-4x}(1+2x),
 \end{aligned} \tag{2.9}$$

for (i) one strange quark and (ii) two strange quarks in the loop, respectively, with  $x = (m_s/m_c)^2$ . Of course,  $\eta_i = 1$  in the absence of strange loop quarks. In Eq. (2.7) the first term proportional to  $g^{\mu\nu}k^2$  contributes to the Pauli interference, while the rest to the  $W$ -exchange or  $W$ -annihilation, where  $k$  is the total four-momentum of the integrated quark pair [9]. More specifically,  $k = p_c + p_q$  for the  $W$ -exchange and  $W$ -annihilation, and  $k = p_c - p_q$  for the Pauli interference. In the heavy quark limit,  $k \rightarrow p_c$  and it is easily seen that (2.7) is reduced to the more familiar form [13]

$$\begin{aligned}
 \mathcal{L}_{\text{NL,nspec}} = \frac{2G_F^2 m_c^2}{\pi} V_{\text{CKM}} \Big\{ & \left( 2c_1 c_2 + \frac{1}{N_c} (c_1^2 + c_2^2) \right) \eta_1 O_{V-A}^d + 2(c_1^2 + c_2^2) \eta_1 T_{V-A}^d \\
 & - \frac{1}{3} N_c \left( c_2 + \frac{1}{N_c} c_1 \right)^2 (\eta_2 O_{V-A}^u - \eta_3 O_{S-P}^u) - \frac{2}{3} c_1^2 (\eta_2 T_{V-A}^u - \eta_3 T_{S-P}^u) \\
 & - \frac{1}{3} N_c \left( c_1 + \frac{1}{N_c} c_2 \right)^2 (\eta_2 O_{V-A}^s - \eta_3 O_{S-P}^s) - \frac{2}{3} c_2^2 (\eta_2 T_{V-A}^s - \eta_3 T_{S-P}^s) \Big\}, \tag{2.10}
 \end{aligned}$$

where use has been made of equations of motion, and

$$\begin{aligned}
O_{V-A}^q &= \bar{c}_L \gamma_\mu q_L \bar{q}_L \gamma^\mu c_L, \\
O_{S-P}^q &= \bar{c}_R q_L \bar{q}_L c_R, \\
T_{V-A}^q &= \bar{c}_L \gamma_\mu t^a q_L \bar{q}_L \gamma^\mu t^a c_L, \\
T_{S-P}^q &= \bar{c}_R t^a q_L \bar{q}_L t^a c_R,
\end{aligned} \tag{2.11}$$

with  $q_{R,L} = (1 \pm \gamma_5)q/2$ .

In analog to the hadronic parameters defined in [13] for the  $B$  meson sector, we can also define four hadronic parameters  $B_1, B_2, \varepsilon_1, \varepsilon_2$  in the charm sector as

$$\begin{aligned}
\frac{1}{2m_{D_q}} \langle D_q | O_{V-A}^q | D_q \rangle &\equiv \frac{f_{D_q}^2 m_{D_q}}{8} B_1, \\
\frac{1}{2m_{D_q}} \langle D_q | T_{V-A}^q | D_q \rangle &\equiv \frac{f_{D_q}^2 m_{D_q}}{8} \varepsilon_1,
\end{aligned} \tag{2.12}$$

and

$$\begin{aligned}
\frac{k^\mu k^\nu}{2m_{D_q}^3} \langle D_q | O_{\mu\nu}^q | D_q \rangle &\equiv \frac{f_{D_q}^2 m_{D_q}}{8} B_2, \\
\frac{k^\mu k^\nu}{2m_{D_q}^3} \langle D_q | T_{\mu\nu}^q | D_q \rangle &\equiv \frac{f_{D_q}^2 m_{D_q}}{8} \varepsilon_2,
\end{aligned} \tag{2.13}$$

for the matrix elements of these four-quark operators between  $D$  meson states. Under the factorization approximation,  $B_i = 1$  and  $\varepsilon_i = 0$  [13].

The destructive Pauli interference in inclusive nonleptonic  $D^+$  and  $D_s^+$  decays and the  $W$ -exchange contribution to  $D^0$  and the  $W$ -annihilation contribution to  $D_s^+$  are

$$\begin{aligned}
\Gamma^{\text{exc}}(D^0) = & -\Gamma_0 \eta_{\text{nspec}} (|V_{cs}|^2 |V_{ud}|^2 + |V_{cd}|^2 |V_{us}|^2) \frac{m_D^2}{m_c^2} (1-x)^2 \\
& \times \left\{ (1 + \frac{1}{2}x) \left[ (\frac{1}{N_c} c_1^2 + 2c_1 c_2 + N_c c_2^2) B_1 + 2c_1^2 \varepsilon_1 \right] \right. \\
& \left. - (1 + 2x) \left[ (\frac{1}{N_c} c_1^2 + 2c_1 c_2 + N_c c_2^2) B_2 + 2c_1^2 \varepsilon_2 \right] \right\} \\
& - \Gamma_0 \eta_{\text{nspec}} |V_{cs}|^2 |V_{us}|^2 \frac{m_D^2}{m_c^2} \sqrt{1-4x} \\
& \times \left\{ (1-x) \left[ (\frac{1}{N_c} c_1^2 + 2c_1 c_2 + N_c c_2^2) B_1 + 2c_1^2 \varepsilon_1 \right] \right. \\
& \left. - (1 + 2x) \left[ (\frac{1}{N_c} c_1^2 + 2c_1 c_2 + N_c c_2^2) B_2 + 2c_1^2 \varepsilon_2 \right] \right\} \\
& - \Gamma_0 \eta_{\text{nspec}} |V_{cd}|^2 |V_{ud}|^2 \frac{m_D^2}{m_c^2} \left\{ (\frac{1}{N_c} c_1^2 + 2c_1 c_2 + N_c c_2^2) (B_1 - B_2) + 2c_1^2 (\varepsilon_1 - \varepsilon_2) \right\},
\end{aligned}$$

$$\begin{aligned}
\Gamma_-^{\text{int}}(D^+) &= \Gamma_0 \eta_{\text{nspec}} |V_{ud}|^2 (|V_{cs}|^2 (1-x)^2 + |V_{cd}|^2) \frac{(\langle p_c \rangle - \langle p_d \rangle)^2}{m_c^2} \\
&\quad \times \left[ (c_1^2 + c_2^2)(B_1 + 6\varepsilon_1) + 6c_1 c_2 B_1 \right], \\
\Gamma^{\text{ann}}(D_s^+) &= -\Gamma_0 \eta_{\text{nspec}} |V_{cs}|^2 |V_{ud}|^2 \frac{m_{D_s}^2}{m_c^2} \left\{ \left( \frac{1}{N_c} c_2^2 + 2c_1 c_2 + N_c c_1^2 \right) (B_1 - B_2) + 2c_2^2 (\varepsilon_1 - \varepsilon_2) \right\} \\
&\quad - \Gamma_0 \eta_{\text{nspec}} |V_{cs}|^2 |V_{us}|^2 \frac{m_{D_s}^2}{m_c^2} (1-x)^2 \left\{ \left( 1 + \frac{1}{2}x \right) \left[ \left( \frac{1}{N_c} c_1^2 + 2c_1 c_2 + N_c c_2^2 \right) B_1 + 2c_1^2 \varepsilon_1 \right] \right. \\
&\quad \left. - (1+2x) \left[ \left( \frac{1}{N_c} c_1^2 + 2c_1 c_2 + N_c c_2^2 \right) B_2 + 2c_1^2 \varepsilon_2 \right] \right\}, \\
\Gamma_-^{\text{int}}(D_s^+) &= \Gamma_0 \eta_{\text{nspec}} |V_{us}|^2 (|V_{cs}|^2 (1-x)^2 + |V_{cd}|^2) \frac{(\langle p_c \rangle - \langle p_s \rangle)^2}{m_c^2} \\
&\quad \times \left[ (c_1^2 + c_2^2)(B_1 + 6\varepsilon_1) + 6c_1 c_2 B_1 \right], \tag{2.14}
\end{aligned}$$

with

$$\Gamma_0 = \frac{G_F^2 m_c^5}{192\pi^3}, \quad \eta_{\text{nspec}} = 16\pi^2 \frac{f_{D_q}^2 m_{D_q}}{m_c^3}. \tag{2.15}$$

In Eq. (2.14),  $\langle p_c \rangle$  and  $\langle p_q \rangle$  ( $q = d, s$ ) are the average momenta of the charmed and light quarks, respectively, in the charmed meson. The sum  $p_c + p_q$  can be effectively substituted by  $m_{D_q}$ , the mass of the charmed meson  $D_q$ . This can be nicely illustrated by the example of  $D_s \rightarrow \tau \bar{\nu}_\tau$  decay with the decay rate:

$$\Gamma(D_s \rightarrow \tau \bar{\nu}_\tau) \simeq \frac{G_F^2 m_\tau^2 f_{D_s}^2 m_{D_s}}{8\pi} |V_{cs}|^2 \left( 1 - \frac{m_\tau^2}{m_{D_s}^2} \right)^2, \tag{2.16}$$

an expression which can be found in the textbook. In the OPE study, the same decay width is represented by

$$\begin{aligned}
\Gamma(D_s \rightarrow \tau \bar{\nu}_\tau) &\simeq \frac{G_F^2}{6\pi} |V_{cs}|^2 \left[ (p_c + p_{\bar{s}})^\mu (p_c + p_{\bar{s}})^\nu - g^{\mu\nu} (p_c + p_{\bar{s}})^2 + \frac{3}{2} g^{\mu\nu} m_\tau^2 \right] \\
&\quad \times \frac{\langle D_s | (\bar{c} \gamma_\mu (1 - \gamma_5) s) (\bar{s} \gamma_\nu (1 - \gamma_5) c) | D_s \rangle}{2m_{D_s}} \left( 1 - \frac{m_\tau^2}{(p_c + p_{\bar{s}})^2} \right)^2. \tag{2.17}
\end{aligned}$$

Comparing the above two expressions, it is clear that  $(p_c + p_{\bar{s}})^2$  is nothing but  $m_{D_s}^2$ . Consequently,  $p_c - p_q$  can be approximated as  $p_{D_q} - 2p_q$  where  $p_q$  could be roughly set as the constituent quark mass  $\sim 350$  MeV. Compared to the naive OPE predictions, it is evident from Eq. (2.14) that the decay widths of  $W$ -exchange and  $W$ -annihilation are enhanced by a factor of  $(m_{D_q}/m_c)^2$ , whereas the Pauli interference is substantially suppressed by a factor of  $(p_{D_q} - 2p_q)^2/m_c^2 \sim 0.5$ .

### III. QCD SUM RULE CALCULATIONS OF FOUR-QUARK MATRIX ELEMENTS

In order to calculate the four-quark matrix elements appearing in the formula of the  $D$  meson lifetimes within the QCD sum rule approach, it is convenient to adopt the following parametrization:

$$\begin{aligned}
\langle D_q(p^D) | O_{\mu\nu}^q | D_q(p^D) \rangle &= (B p_\mu^D p_\nu^D + \delta B g_{\mu\nu} m_{D_q}^2) \frac{f_{D_q}^2}{4}, \\
\langle D_q(p^D) | T_{\mu\nu}^q | D_q(p^D) \rangle &= (\varepsilon p_\mu^D p_\nu^D + \delta \varepsilon g_{\mu\nu} m_{D_q}^2) \frac{f_{D_q}^2}{4},
\end{aligned} \tag{3.1}$$

where the relations between  $B, \delta B, \varepsilon, \delta \varepsilon$  and the parameters  $B_{1,2}, \varepsilon_{1,2}$  defined in Eqs. (2.12) and (2.13) are

$$\begin{aligned}
B_1 &= B + 4\delta B, \quad B_2 = B + \delta B, \\
\varepsilon_1 &= \varepsilon + 4\delta \varepsilon, \quad \varepsilon_2 = \varepsilon + \delta \varepsilon.
\end{aligned} \tag{3.2}$$

Unlike the  $B$  meson case, the study of the  $D$  meson is preferred to begin with the full theory directly for several reasons: (1) In the QCD sum rule study of the full theory, the working Borel window of the  $D$  meson case is about  $2.0 \text{ GeV}^2 < M^2 < 3.0 \text{ GeV}^2$ . Hence, the extraction of relevant 4-quark matrix elements can be obtained directly at the scale  $\sim m_c$ . (2) Since the physical quantities expanded in  $1/m_c$  will converge slowly due to the fact that  $m_c$  is not heavy enough, it becomes unnecessary to work with the effective theory at the outset. (3) It is customary in the literature to evolve the hadronic matrix elements down to the confinement scale, say  $\mu_h \sim 500 \text{ MeV}$ , in order to apply the vacuum insertion hypothesis. However, as emphasized in Ref. [14], we shall avoid evaluating the matrix elements in such a low scale because  $\alpha(\mu_h)$  is of order unity at this scale and large radiative corrections cannot be entirely grouped into the Wilson coefficients.

We consider the following three-point correlation functions

$$\begin{aligned}
\Pi_{\mu\nu}^O(p, p') &= i^2 \int dx dy e^{ipx - ip'y} \langle 0 | T \{ [\bar{q}(x) i\gamma_5 c(x)] O_{\mu\nu}^q(0) [\bar{q}(y) i\gamma_5 c(y)]^\dagger \} | 0 \rangle, \\
\Pi_{\mu\nu}^T(p, p') &= i^2 \int dx dy e^{ipx - ip'y} \langle 0 | T \{ [\bar{q}(x) i\gamma_5 c(x)] T_{\mu\nu}^q(0) [\bar{q}(y) i\gamma_5 c(y)]^\dagger \} | 0 \rangle.
\end{aligned} \tag{3.3}$$

The sum rule calculation gives

$$\begin{aligned}
&\frac{B p_\mu p'_\nu + \delta B p \cdot p' g_{\mu\nu}}{(p^2 - m_{D_q}^2)(p'^2 - m_{D_q}^2)} \left( \frac{f_{D_q} m_{D_q}^2}{m_c + m_q} \right)^2 \frac{f_{D_q}^2}{4} \\
&\simeq \frac{1}{4} p_\mu p'_\nu \left\{ \frac{3}{8\pi^2} \int_{m_c^2}^{s_0} ds \frac{1}{s - p^2} \left[ m_c \left( 1 - \frac{m_c^2}{s} \right)^2 + m_q \left( 1 - \frac{m_c^4}{s^2} \right) \right] + \frac{\langle \bar{q}q \rangle}{p^2 - m_c^2} \left( 1 + \frac{m_c m_q}{2(p^2 - m_c^2)} \right) \right. \\
&+ \frac{\langle g_s^2 G^2 \rangle}{48\pi^2 m_c} \left( \frac{1}{p^2} - \frac{1}{p^2 - m_c^2} \right) - \frac{m_c^2 \langle g_s \bar{q} \sigma G q \rangle}{2(p^2 - m_c^2)^3} \left( 1 + \frac{m_q}{4m_c} + \frac{m_c m_q}{8(p^2 - m_c^2)} \right) \left. \right\}^2 \\
&+ g_{\mu\nu} p \cdot p' \times \mathcal{O}(\text{dimension } 8),
\end{aligned} \tag{3.4}$$

and

$$\begin{aligned}
&\frac{\varepsilon p_\mu p'_\nu + \delta \varepsilon p \cdot p' g_{\mu\nu}}{(p^2 - m_{D_q}^2)(p'^2 - m_{D_q}^2)} \left( \frac{f_{D_q} m_{D_q}^2}{m_c + m_q} \right)^2 \frac{f_{D_q}^2}{4} = -\frac{1}{3} (g_{\mu\nu} p \cdot p' - p_\mu p'_\nu) m_c^3 \\
&\times \left\{ \frac{\langle g_s^2 G^2 \rangle}{(32\pi^2)^2} \left[ \int_{m_c^2}^{s_0} ds \int_{m_c^2}^{s_0} ds' \frac{1}{(s - p^2)(s' - p'^2)} \frac{1}{s^2 s'^2} [m_c(s + s' - m_c^2) - 2m_q(2s + 2s' - m_c^2)] \right. \right.
\end{aligned}$$



$$\begin{aligned}
& +2\frac{m_q}{m_c^2}\left(\frac{\ln[(m_c^2 - p'^2)/(\mu m_c)]}{p'^2 - m_c^2}\int_{m_c^2}^{s_0} ds \frac{2s - 3m_c^2}{s^2(s - p^2)} + \frac{\ln[(m_c^2 - p^2)/(\mu m_c)]}{p^2 - m_c^2}\int_{m_c^2}^{s_0} ds' \frac{2s' - 3m_c^2}{s'^2(s' - p^2)}\right) \\
& - \frac{\langle g_s \bar{q} \sigma \cdot G q \rangle}{128\pi^2} \left[ \int_{m_c^2}^{s_0} ds \int_{m_c^2}^{s_0} ds' \frac{1}{(s - p^2)(s' - p'^2)} \left( \frac{1}{s'^2} \delta(s - m_c^2) + \frac{1}{s^2} \delta(s' - m_c^2) \right) \left( 1 - \frac{3m_q}{m_c} \right) \right. \\
& \left. - 4\frac{m_q}{m_c^3} \left( \frac{\ln[(m_c^2 - p'^2)/(\mu m_c)]}{p'^2 - m_c^2} \frac{1}{p^2 - m_c^2} + \frac{\ln[(m_c^2 - p^2)/(\mu m_c)]}{p^2 - m_c^2} \frac{1}{p'^2 - m_c^2} \right) \right] \Big\} \\
& + \mathcal{O}(\text{dimension } 6), \tag{3.5}
\end{aligned}$$

where  $\langle \dots \rangle$  stands for  $\langle 0 | \dots | 0 \rangle$  and

$$\langle 0 | \bar{q} i \gamma_5 c | D_q \rangle = \frac{f_{D_q} m_{D_q}^2}{m_c + m_q}. \tag{3.6}$$

Here we have used the factorization (or vacuum insertion) approximation to estimate the four-quark condensate. However, since  $\delta B$  does not receive four-quark operator contributions under the factorization approximation, the contribution from non-vacuum intermediate states may not be negligible<sup>1</sup>. We would like to remind readers that the ratio of  $\tau(D_s^+)/\tau(D^0)$  is quite sensitive to  $\delta B$ . We have estimated the four-gluon condensate contribution to  $\delta B$  and found that the enhancement of  $\delta B$  due to the four-gluon condensate is less  $10^{-3}$  and thus can be neglected. After performing the double Borel transformations [14],  $p^2 \rightarrow M^2$  and  $p'^2 \rightarrow M'^2$ , on the above sum rules and letting  $M^2 = M'^2$ , we obtain

$$\begin{aligned}
B = & 4 \left( \frac{m_c + m_q}{f_{D_q}^2 m_{D_q}^2} \right)^2 e^{2m_{D_q}^2/M^2} \left\{ \frac{3}{8\pi^2} \int_{m_c^2}^{s_0} ds e^{-s^2/M^2} \left[ m_c \left( 1 - \frac{m_c^2}{s} \right)^2 + m_q \left( 1 - \frac{m_c^4}{s^2} \right) \right] \right. \\
& - \langle \bar{q} q \rangle \left( 1 - \frac{m_c m_q}{2M^2} \right) e^{-m_c^2/M^2} - \frac{\langle g_s^2 G^2 \rangle}{48\pi^2 m_c} \left( 1 - e^{-m_c^2/M^2} \right) \\
& \left. + \frac{m_c^2 \langle g_s \bar{q} \sigma \cdot G q \rangle}{4M^4} \left( 1 + \frac{m_q}{4m_c} - \frac{m_c m_q}{24M^2} \right) e^{-m_c^2/M^2} \right\}^2, \tag{3.8}
\end{aligned}$$

$$\delta B \approx 0,$$

and

$$\varepsilon = -\delta\varepsilon = \left( \frac{m_c + m_q}{f_{D_q}^2 m_{D_q}^2} \right)^2 \frac{4}{3} m_c^3 e^{2m_{D_q}^2/M^2} \int_{m_c^2}^{s_0} ds \int_{m_c^2}^{s_0} ds' e^{-(s+s')/M^2}$$

---

<sup>1</sup>Up to dimension six,  $\delta B$  is given by

$$\frac{\delta B p \cdot p'}{(p^2 - m_{D_q}^2)(p'^2 - m_{D_q}^2)} \left( \frac{f_{D_q}^2 m_{D_q}^2}{m_c + m_q} \right)^2 = \frac{m_c^2}{4} \langle \bar{q} \gamma_\mu (1 - \gamma_5) q \bar{q} \gamma^\mu (1 - \gamma_5) q \rangle \frac{1}{(p^2 - m_c^2)(p'^2 - m_c^2)}, \tag{3.7}$$

which obviously vanishes under the factorization approximation. At the confinement scale  $\sim 500$  MeV, the nonfactorizable contribution due to the four-quark condensate was shown to be sizable in [9]. As a result, the lifetime ratio of  $\tau(D_s^+)/\tau(D^0) \sim 1.24$  obtained in [9] is much larger than previous estimates.

$$\begin{aligned}
& \left\{ \frac{\langle g_s^2 G^2 \rangle}{(32\pi^2)^2} \frac{1}{s^2 s'^2} \left[ m_c(s + s' - m_c^2) - 2m_q(2s + 2s' - m_c^2) \right. \right. \\
& + 4m_q(2s - 3m_c^2)s'\delta(s' - m_c^2) \left( \gamma + \ln \frac{(\mu m_c)}{M^2} \right) \left. \right] \\
& - \frac{\langle g_s \bar{q} \sigma \cdot G q \rangle}{128\pi^2} \left[ \left( \frac{1}{s'^2} \delta(s - m_c^2) + \frac{1}{s^2} \delta(s' - m_c^2) \right) \left( 1 - \frac{3m_q}{m_c} \right) \right. \\
& \left. \left. + \frac{8m_q}{m_c^3} \delta(s - m_c^2) \delta(s' - m_c^2) \left( \gamma + \ln \frac{(\mu m_c)}{M^2} \right) \right] \right\}, \tag{3.9}
\end{aligned}$$

where  $\gamma$  is the Euler's constant.

For numerical estimates of  $B$  and  $\varepsilon$ , we shall use the following values of parameters: <sup>2</sup>  $f_{D_{u,d}} = 170 \pm 10$  MeV,  $f_{D_s} = 210 \pm 10$  MeV,  $m_u = m_d = 0$ ,  $m_s = 125 \pm 25$  MeV,  $m_c = 1.40 \pm 0.05$  GeV,  $s(D_{u,d}) = 6$  GeV<sup>2</sup>,  $s(D_s) = 6.5$  GeV<sup>2</sup>, and [14]

$$\begin{aligned}
\langle \bar{u}u \rangle_{\mu=1 \text{ GeV}} &= \langle \bar{d}d \rangle_{\mu=1 \text{ GeV}} = -(240 \pm 20 \text{ MeV})^3, \\
\langle \bar{s}s \rangle &= 0.8 \times \langle \bar{u}u \rangle, \\
\langle \alpha_s G^2 \rangle_{\mu=1 \text{ GeV}} &= 0.0377 \text{ GeV}^4, \\
\langle \bar{q}g_s \sigma \cdot G q \rangle &= (0.8 \text{ GeV}^2) \times \langle \bar{q}q \rangle. \tag{3.10}
\end{aligned}$$

Note that in the sum rule study,  $m_c$  is the current quark mass normalized at  $\mu^2 = -m_c^2$ .

To further improve the quality of the sum-rule results, we rescale the nonperturbative quantities to the scale of the Borel mass  $M$ .

$$\begin{aligned}
f_{D_q}(M) &= f_{D_q}(m_c) \left( \frac{\alpha_s(M)}{\alpha_s(m_c)} \right)^{-2/\beta_0}, \\
\langle \bar{q}q \rangle_M &= \langle \bar{q}q \rangle_\mu \cdot \left( \frac{\alpha_s(M)}{\alpha_s(\mu)} \right)^{-4/\beta_0}, \\
\langle g_s \bar{q} \sigma \cdot G q \rangle_M &= \langle g_s \bar{q} \sigma \cdot G q \rangle_\mu \cdot \left( \frac{\alpha_s(M)}{\alpha_s(\mu)} \right)^{2/(3\beta_0)}, \\
\langle \alpha_s G^2 \rangle_M &= \langle \alpha_s G^2 \rangle_\mu, \tag{3.11}
\end{aligned}$$

where  $\beta_0 = \frac{11}{3} N_c - \frac{2}{3} n_f$  is the leading-order expression of the  $\beta$ -function with  $n_f$  being the number of light quark flavors.

Let us explain the results obtained in Eq. (3.8) for the parameter  $B$  and Eq. (3.9) for  $\varepsilon$ . Eq. (3.8) can be approximately factorized as a product of two two-point  $f_{D_q}$  sum rules. As a result,  $B \approx 1$ . To the order of dimension-five, the main contributions to the OPE series of  $\varepsilon$  are depicted in Fig. 2, where we have neglected the dimension-six four-quark condensate of the type  $\langle \bar{q} \Gamma \lambda^a q \bar{q} \Gamma \lambda^a q \rangle$  since its contribution is much less than that from dimension-five or

---

<sup>2</sup>It is known that the charmed quark mass used in the sum-rule studies is smaller than the pole mass shown below. Likewise, the sum-rule decay constants  $f_D$  and  $f_{D_s}$  are slightly smaller the values employed in Sec. IV.

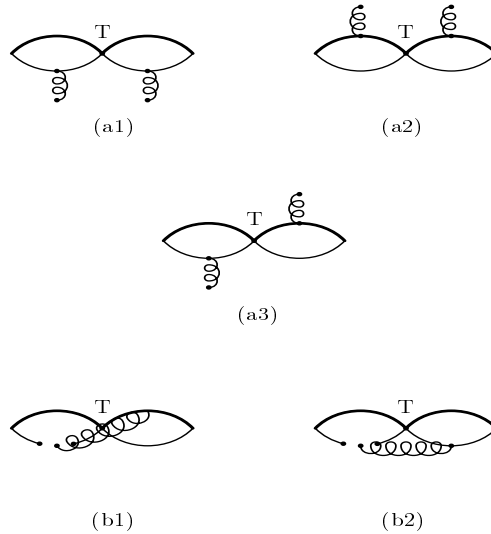


FIG. 2. The main diagrams contributing to the OPE series of  $\varepsilon$  in Eq. (3.9): (a1)–(a3) the gluon condensates, and (b1)–(b2) the quark-gluon mixed condensates. The charmed quark is denoted by the heavy line.

dimension-four condensates. The numerical result of  $\varepsilon$  ( $= -\delta\varepsilon$ ) is shown in Fig. 3. Within the Borel window  $2.0 \text{ GeV}^2 < M^2 < 3.0 \text{ GeV}^2$ , we obtain  $\varepsilon(D^{0,+}) = -\delta\varepsilon(D^{0,+}) = 0.015 \pm 0.010$  and  $\varepsilon(D_s^+) = -\delta\varepsilon(D_s^+) = 0.015_{-0.010}^{+0.015}$ , where the error comes partially from the uncertainties of input parameters. Consequently,  $B_{1,2}$  and  $\varepsilon_{1,2}$  are numerically given by

$$B_1 = B_2 \approx 1, \quad \varepsilon_1(D^{0,+}) = -0.045 \pm 0.030, \quad \varepsilon_1(D_s^+) = -0.045_{-0.030}^{+0.045}, \quad \varepsilon_2 = 0. \quad (3.12)$$

Since the sum rule calculation is built upon the quark-hadron duality hypothesis, it is difficult to estimate the intrinsic errors in this approach. However, if the OPE series is extended to higher dimension operator terms, then the errors will be improved. Moreover, it is desirable to evaluate the non-vacuum intermediate state contributions to  $\delta B = 3(B_1 - B_2)$  as the ratio of  $\tau(D_s^+)/\tau(D^0)$  is quite sensitive to  $\delta B$ . Even if  $\delta B$  deviates from zero by a small amount, say 0.005, the ratio  $\tau(D_s^+)/\tau(D^0)$  will be enhanced by 6%.

#### IV. RESULTS AND DISCUSSIONS

The total decay width of the charmed meson is given by

$$\Gamma(D) = \Gamma_{\text{NL,spec}} + \Gamma_{\text{NL,nspec}} + \Gamma_{\text{SL}} + \Gamma_{\text{lep}}, \quad (4.1)$$

where  $\Gamma_{\text{NL,spec}}$  and  $\Gamma_{\text{NL,nspec}}$  denote nonleptonic decay widths [cf. Eqs. (2.1) and (2.7)] due to spectator and nonspectator contributions, respectively,  $\Gamma_{\text{SL}}$  [see Eq. (2.2)] and  $\Gamma_{\text{lep}}$  the

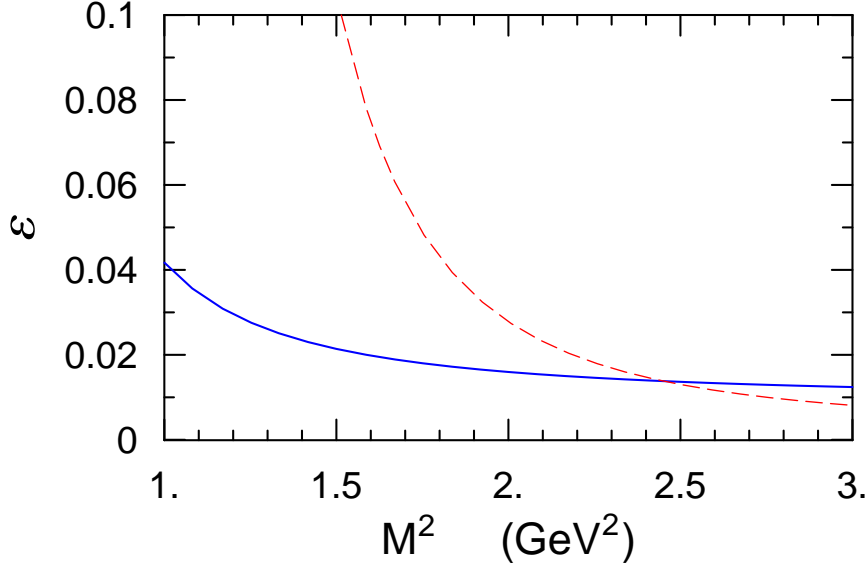


FIG. 3.  $\varepsilon$  ( $= -\delta\varepsilon$ ) as a function of the Borel mass squared  $M^2$ . The solid and dashed curves are for  $\varepsilon(D^{0,+})$  and  $\varepsilon(D_s^+)$ , respectively. Here we have used  $f_D = 170$  MeV,  $f_{D_s} = 210$  MeV,  $m_c = 1.40$  GeV,  $m_s = 125$  MeV,  $s(D^{0,+}) = 6$  GeV<sup>2</sup>,  $s(D_s) = 6.5$  GeV<sup>2</sup>,  $\langle \bar{q}q \rangle_{\mu=1 \text{ GeV}} = -(240 \text{ MeV})^3$ , and Eq. (3.10).

semileptonic and pure leptonic decay widths, respectively. In units of  $\Gamma_0 = G_F^2 m_c^5 / (192\pi^3)$ , we obtain  $\Gamma_{\text{NL,spec}} = 4.84\Gamma_0$ ,  $\Gamma_{\text{SL}} = 1.02\Gamma_0$  and  $\Gamma_{\text{lep}}(D_s^+ \rightarrow \tau\bar{\nu}_\tau + \mu\bar{\nu}_\mu) = 0.169\Gamma_0$  for  $m_c = 1.65$  GeV,  $m_s = 125$  MeV,  $c_1(m_c) = 1.30$  and  $c_2(m_c) = -0.57$ .

If the momentum of the spectator quark in the  $D^+$  meson is neglected, the destructive Pauli interference in  $D^+$  decay is found to be  $\Gamma_-^{\text{int}}(D^+) = -8.5\Gamma_0$ , which largely overcomes the  $c$ -quark decay rate  $\Gamma_{\text{NL,spec}}$ . Consequently,  $\Gamma_{\text{tot}}(D^+)$  becomes negative, which is of course of no sense. This indicates that it is mandatory to invoke the spectator quark to suppress the Pauli interference effect [see Eq. (2.14)]. On the contrary, the spectator quark's momentum in the charmed meson will enhance the  $W$ -exchange or  $W$ -annihilation contribution. Since the decay width of  $D^+$  involves a large cancellation between two terms, it is very sensitive to the parameters  $m_c$ ,  $f_D$  and  $\langle p_q \rangle$ . For  $f_D = 190$  MeV and  $\langle p_q \rangle = 350$  MeV, we found that the pole mass  $m_c$  is preferred to be a bit larger. We shall use  $m_c = 1.65$  GeV for calculation.

We next proceed to compute the non-spectator effects using Eqs. (2.14) and (3.12) and obtain

$$\begin{aligned}
\Gamma^{\text{exc}}(D^0) &= (0.46 \pm 0.30)\Gamma_0, \\
\Gamma_-^{\text{int}}(D^+) &= -(3.29 \pm 0.40)\Gamma_0, \\
\Gamma^{\text{ann}}(D_s^+) &= (0.19 \pm 0.13)\Gamma_0, \\
\Gamma_-^{\text{int}}(D_s^+) &= -(0.35 \pm 0.05)\Gamma_0,
\end{aligned} \tag{4.2}$$

where the errors come from the uncertainty of  $\varepsilon_1$ , and use has been made of  $f_{D_s} = 240$  MeV. Collecting all the contributions, we find

$$\tau(D^0) = 0.38 \text{ ps}, \quad \tau(D^+) = 0.96 \text{ ps}, \quad \tau(D_s^+) = 0.41 \text{ ps}. \quad (4.3)$$

It is clear from our calculations that the lifetime of  $D_s^+$  is longer than that of  $D^0$  because the Cabibbo-allowed nonleptonic annihilation and leptonic contributions to  $\Gamma(D_s^+)$  are compensated by the Cabibbo-suppressed Pauli interference. We also see that the predicted absolute charmed meson lifetimes are in general too small compared to experiments [4]:

$$\tau(D^0) = (0.415 \pm 0.004) \text{ ps}, \quad \tau(D^+) = (1.057 \pm 0.015) \text{ ps}, \quad (4.4)$$

and

$$\tau(D_s^+) = \begin{cases} (0.518 \pm 0.014 \pm 0.007) \text{ ps} & \text{(E791) [3],} \\ (0.4863 \pm 0.015 \pm 0.005) \text{ ps} & \text{(CLEO) [5].} \end{cases} \quad (4.5)$$

By contrast, the calculated lifetimes of  $B$  and  $\Lambda_b$  hadrons based on heavy quark expansion are too large compared to the data (see e.g. [12]).

The charm lifetime ratios followed from Eq. (4.3) are

$$\begin{aligned} \frac{\tau(D^+)}{\tau(D^0)} &\simeq 2.56 \pm 0.52, \\ \frac{\tau(D_s^+)}{\tau(D^0)} &\simeq 1.08 \pm 0.04. \end{aligned} \quad (4.6)$$

Although the lifetime ratio  $\tau(D^+)/\tau(D^0)$  is in accordance with experiment, the predicted ratio for  $\tau(D_s^+)/\tau(D^0)$ , which is insensitive to the value of  $m_c$ , is larger than previous theoretical estimates [1,2] but still smaller than recent measurements. Nevertheless, this lifetime ratio could get enhanced if non-vacuum intermediate states contribute sizably to the four quark condensate so that  $\delta B$  is nonzero. It is worth remarking that if the nonzero momentum of the spectator quark is neglected, then the ratio  $\tau(D_s^+)/\tau(D^0)$  will be enhanced to 1.11. However, as stressed in passing, it is meaningless to have a negative lifetime for the  $D^+$ .

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